Unified inverse correspondence for DLE-logics

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Driven by the "insight that almost all completeness proofs can be reinterpreted as definability results [...] and that also correspondence theory is a kind of definability theory", Kracht [21] developed the theory of internal description, sometimes referred to as *inverse correspondence* [1]. This theory can be regarded as converse to Sahlqvist correspondence [22]; indeed, it syntactically identifies a class of first order formulas, each of which is the first order correspondent of some modal formula, and provides an effective procedure for computing such modal formula.

Goranko and Vakarelov extended Sahlqvist theory to the class of *polyadic Sahlqvist formulas* [13], also referred to as *inductive formulae* [14]. In [20], Kikot extends Kracht's result to inductive formulae, by syntactically characterizing a class of formulas in the first order language of Kripke frames for classical normal modal logic which correspond to inductive formulas in classical modal logic.

During the last decade, a line of research was developed which focuses on the order-theoretic underpinning of Sahlqvist theory, thus allowing for the generalisations of this theory from classical modal logic to wide classes of nonclassical logics. This shift from a model-theoretic to an algebraic perspective made it possible to uniformly define the class of Sahlqvist and inductive formulas/inequalities for a broad spectrum of logical languages, based on the order-theoretic properties of the algebraic interpretations of the logical connectives in each language, and to extend the algorithm SQEMA, for computing the first order correspondents of inductive formulas of classical modal logic [7], to the algorithm ALBA [8, 9], performing the same task as SQEMA for this spectrum of nonclassical languages which includes the *LE-logics*, i.e. those logics the algebraic semantics of which is given by varieties of normal/regular lattice expansions (LEs), and their expansions with fixed points [6, 3]. This very high level of generality has made it possible to extend the benefits of correspondence and canonicity results to many well known logical systems such as bi-intuitionistic (modal) logic, the Lambek-Grishin calculus [19], and the multiplicative-additive fragment of linear logic [12]. Moreover, it has also allowed for several developments and connections among the meta-theories of various logical frameworks, examples of which are a general perspective on Gödel-McKinsey-Tarski translations and correspondence/canonicity transfer results [10, 11], systematic connections among different relational semantics of a given logic [5], and systematic connections between correspondence-theoretic results and the proof-theoretic behaviour of logical frameworks [16, 17, 2, 18, 15].

While many generalizations of Sahlqvist correspondence theory have been developed in recent times, no generalizations of Kracht's theory of inverse correspondence have been investigated yet since Kikot's result. Our proposed talk presents the results of [4] which start to fill this gap, by generalizing Kikot's result from classical normal modal logic to all normal *DLE-logics*, i.e. those logics the algebraic semantics of which is given by varieties of normal distributive lattice expansions (DLEs). In particular, we introduce an *inverse correspondence* algorithm targeting inductive inequalities in any DLE-signature.

Key to this extension is the possibility to reformulate the main engine of Kracht's result in the algebraic environment of unified correspondence [6] so as to exploit the language and algorithmic tools developed there, which work across signatures and relational semantics. Indeed, to achieve this objective, we approach the problem from an exclusively order theoretic perspective by making use of a slight extension of ALBA's language and rules.

The proof-strategy adopted to achieve this result is different from Kikot's. Indeed, rather than relaxing the definition of Kracht's formula, which is given only in terms of forward-looking restricted quantifiers, we

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start by generalizing to the setting of DLE-logics the fact, well-known from classical modal logic, that inductive formulas are semantically equivalent to (a certain proper subclass) of scattered *very simple Sahlqvist* formulas in the language of *tense logic*. Accordingly, for every DLE-language $\mathcal{L}$, we syntactically characterize the class K of very simple Sahlqvist $\mathcal{L}^*$-inequalities (where $\mathcal{L}^*$ is the language expansion of $\mathcal{L}$ obtained by closing the signature of $\mathcal{L}$ under the residuals of each connective in $\mathcal{L}$) which are semantically equivalent to inductive $\mathcal{L}$-inequalities. Then, we syntactically characterize the class of formulas in the ALBA-language, referred to as *Kracht's formulas* (which can be readily translated into first-order formulas of a given frame correspondence language) which target the subclass K, by allowing for the use of *backward-looking* restricted quantifiers. Finally, we show that each Kracht's formula in the ALBA-language can be effectively and equivalently transformed into the ALBA-output of an $\mathcal{L}^*$-inequality in K.

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