Algorithmic correspondence and analytic rules for (D)LE logics

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A core line of research in structural proof theory focuses on the algorithmic or semi-algorithmic generation of analytic rules (we refer to [1] for a detailed survey of the relevant literature).

In [15, 12, 13, 14], a class of first order formulas, referred to as *(co-)geometric formulas*, is identified and used for effectively generating analytic rules extending a basic relational labelled calculus for classical and intuitionistic modal logic. For any (co-)geometric first order formula α, the procedure generates a rule r; moreover, if α is the first order correspondent of a modal formula φ then r equivalently captures φ.

In [2, 10, 11], a class of formulas in the signature of the full Lambek calculus is identified, in the context of a syntactic hierarchy (known as the *proof-theoretic substructural hierarchy*), and an algorithm is introduced for generating analytic rules of a Gentzen-style sequent calculus (resp. hypersequent calculus). This approach was further extended in [3] (generalizing a result for tense modal logic in [9]) to characterize the expressive power of given but not fixed display calculi (from formulas of a given shape to analytic structural rules, and vice versa whenever the calculus satisfies additional conditions).

In [8], a characterization is introduced, analogous to the one of [3] and generalizing [9], of the expressive power of (properly) display calculi, in the context of arbitrary normal (D)LE-logics, i.e. those logics algebraically captured by varieties of normal (distributive) lattice expansions. This characterization is achieved via a systematic connection established between analytic rule-generation and algorithmic correspondence theory [4, 5, 6]. In particular, the same algorithm (ALBA) introduced for generating the first order correspondents of inductive (D)LE-inequalities is used in [8] for generating analytic structural rules of proper display calculi, and the syntactic class of *analytic inductive (D)LE-inequalities* is characterized as those giving rise to properly displayable axiomatic extensions of the basic normal (D)LE-logics.

The contribution discussed in the present talk extends the insights about the systematic connection between algorithmic rule-generation and correspondence theory developed in [8] to relational labelled calculi. Firstly, we use the language of ALBA to encode relational information in a uniform way for any (D)LE-signature; this makes it possible to uniformly design labelled calculi for every basic (D)LE-logic, in which the logical rules encode the behaviour characteristic to each (D)LE-connective in any signature; secondly, we generalize the algorithm MASSA, introduced in [7], to any (D)LE-signature. The general algorithm takes analytic inductive inequalities in input, and outputs (a set of) equivalent analytic rules of a relational labelled calculus. We also show that this algorithm succeeds on all analytic inductive inequalities of any (D)LE-signature.

An important difference between the present algorithmic rule-generation method and Negri's method is that the present method takes *propositional ((D)LE-)inequalities* in input, and, if the input inequality is analytic inductive, it computes its equivalent analytic rule *directly* from the input inequality, via a computation which incorporates the effective generation of its first-order correspondent, whereas Negri's method starts from geometric implications in the first-order frame correspondence language, and generates rules which are equivalent to those modal formulas which are assumed to have a first-order correspondent which is (logically equivalent to) a geometric implication.

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