Comparison of tabular intermediate logics

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A (propositional) *intermediate logic* is a logic being between intuitionistic logic and classical logic (see e.g. [1, 5] for needed definitions). An intermediate logic is *tabular* if it possesses a semantics given by a finite frame P (here just a finite poset). In such a case the logic is denoted by $L(P)$.

We study the complexity of the following decision problem:

Problem: **Int-Log-Contain**;
Instance: Two finite frames P and Q;
Question: Does $L(P) \subseteq L(Q)$?

Changing the sign of containment for the sign of equality gives us the **Int-Log-Equal** problem. Here is our main result.

Theorem 1. *The problems* **Int-Log-Contain** *and* **Int-Log-Equal** *are* NP*-complete.*

Let us sketch the proof. A frame is *rooted* if, as a poset, it has a minimal element. The following equivalence follows from the Jankov-de Jongh theorem [2, 4] and the fact that the operations of taking generated subframes and p-morphic images preserve the satisfaction of intuitionistic formulas.

Proposition 2. *Let P and Q be finite frames. Then $L(P) \subseteq L(Q)$ iff every rooted generated subframe of Q if a p-morphic image of a rooted generated subframe of P.*

This shows that **Int-Log-Contain** is in the NP complexity class. It also shows that the following problem is trivially reducible to **Int-Log-Contain**.

Problem: **p-Image-Gen-Sub**;
Instance: Two finite rooted frames P and Q;
Question: Does there exist a surjective p-morfizm from a generated subframe of P onto Q?

We prove that the listed problems are NP-hard by presenting a polynomial-time reduction of the known NP-complete problem **Monotone-Not-All-Equal-3-Sat** [3] into **p-Image-Gen-Sub**.

Lastly, we infer that the following related problem is also NP-complete.

Problem: **p-Image**;
Instance: Two finite rooted frames P and Q;
Question: Does there exist a surjective p-morfizm from P onto Q?

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