Remarks on enriched protomodularity

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This work was motivated by the study of the categories OrdGp of preordered groups and OrdAb of preordered abelian groups, enriched in the category Ord of preordered sets. Note that OrdGp differs from the category of the internal groups in Ord, since the inversion morphism of the group structure is not necessarily monotone. As a consequence, many of the nice algebraic properties of (abelian) groups fail to hold in that context.

In this talk we focus on the algebraic property of protomodularity, that is, on the validity of the Split Short Five Lemma, and on a possible enriched version of it. Although the category of (abelian) groups is protomodular, OrdGp and OrdAb are not. It is as if the preorder structure works against protomodularity. However, the enriched preordered structure on morphisms does work in favour of protomodularity in the following sense. Having in mind the role of comma objects in the enriched context, we consider some of the characteristic properties of protomodularity with respect to comma objects instead of pullbacks. We show that the equivalence between protomodularity and certain properties on pullbacks also holds when replacing conveniently pullbacks by comma objects in any finitely complete category enriched in Ord. We show that OrdAb gives an example of such an enriched protomodular category.

References

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