On the double category of coalgebras

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Classically, two states of coalgebras of the same functor are behaviourally equivalent whenever they are identifiable by morphisms of coalgebras that share the same codomain. However, for quantitative systems it is often more reasonable to consider states with "close behaviour" instead, which leads to the notion of *behavioural distance* [1] and of metric bisimulation [4]. We show that the latter notion is captured by the first one; that is, the notion of similarity provided by a lax extension corresponding to a class of monotone predicate liftings coincides with the notion of behavioural distance provided by the lifting associated with the same class of predicate liftings. This is the missing link mentioned in [3] that makes is possible to incorporate the approach to similarity via lax extensions in the categorical frameworks described in [2] and [3]. In fact, we describe this connection at a high level of generality and argue that a natural double category of coalgebras for a lax double functor provides a suitable context to reason coalgebraically about various notions of *indistinguishability*. From this point of view we also recover the results from [5] and obtain a new result for (quasi) uniform spaces which complements the expressivity result for uniform spaces obtained in [3].

In this talk we report on joint work with Sergey Goncharov, Pedro Nora, Lutz Schröder and Paul Wild (Friedrich-Alexander-Universität Erlangen-Nürnberg).

References

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