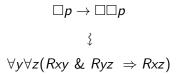
Unified inverse correspondence for DLE-Logics

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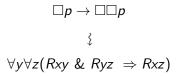
The problem

Correspondence



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Inverse correspondence

(Partial) solution in the classical setting

Kracht formulae

$$\chi(x_0) = (\forall x_1 \rhd x_{i_1}) \dots (\forall x_n \rhd x_{i_n}) (Q_1 y_1 \rhd z_{j_1}) \dots (Q_m y_m \rhd z_{j_m})\beta,$$

where

- ▶ *Q_i* are quantifiers,
- \triangleright x_0 is the only free variable,
- β is a DNF of atoms involving variables in $\{x_1, \ldots, x_n\}$

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Sahlqvist formulae ++++ Kracht formulae

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Generalizing to inductive - Problems

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has FO correspondent

$$\forall x \exists y (xRy \& \forall z (yR^2z \Rightarrow \exists w (wRz \& wRx \& xRw)))$$

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A (D)LE language $\mathcal{F}(\mathcal{F}, \mathcal{G})$ is extended to

 $\varphi ::= \mathbf{j} \mid \mathbf{m} \mid p \mid \varphi \land \varphi \mid \varphi \lor \varphi \mid f(\varphi, \dots, \varphi) \mid g(\varphi, \dots, \varphi),$

where $p \in AtProp$, $\mathbf{j} \in NOM$, $\mathbf{m} \in CONOM$, and $f \in \mathcal{F}^*, g \in \mathcal{G}^*$.

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 $\begin{array}{rcl} \mbox{Algebraic interpretation} & \rightsquigarrow & \mbox{Agnostic wrt. semantics} \\ \mbox{Arbitrary} (\mathcal{F}, \mathcal{G}) & \rightsquigarrow & \mbox{Agnostic wrt. signature} \end{array}$

In a distributive lattice $\mathbb A,$ there are isomorphisms κ and $\lambda,$

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The language is further extended with literals $\kappa(\mathbf{j})$ and $\lambda(\mathbf{m})$.

$$\begin{array}{rcl} \mathsf{NL} &=& \mathsf{NOM} & \cup & \{\lambda(\mathbf{m}) : \mathbf{m} \in \mathsf{CONOM}\}, \\ \mathsf{CNL} &=& \mathsf{CONOM} & \cup & \{\kappa(\mathbf{j}) : \mathbf{j} \in \mathsf{NOM}\}. \end{array}$$

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We write $\rho \mathbf{j}$ to denote $\kappa(\mathbf{j})$, and $\rho \mathbf{m}$ for $\lambda(\mathbf{m})$.

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$$NL = NOM \cup \{\lambda(\mathbf{m}) : \mathbf{m} \in CONOM\},\$$

$$CNL = CONOM \cup \{\kappa(\mathbf{j}) : \mathbf{j} \in NOM\}.$$

We write $\rho \mathbf{j}$ to denote $\kappa(\mathbf{j})$, and $\rho \mathbf{m}$ for $\lambda(\mathbf{m})$.

Lemma For all **j**, **m**, and φ ,

 $\mathbf{j} \leq \varphi \quad iff \quad \varphi \nleq \kappa(\mathbf{j}) \qquad \qquad \varphi \leq \mathbf{m} \quad iff \quad \lambda(\mathbf{m}) \nleq \varphi$

Flat and restricting inequalities

In the classical setting FO-atomic are equivalent to inequalities

xRy	$x \leq \Diamond y$	iff	$\Box y^c \leq x^c$			
	$y \leq \blacklozenge x$	iff	$\blacksquare x^c \le y^c$			
x = y	$x \leq y$	iff	$y^c \leq x^c$	iff	$x \nleq y^c$	
x = y = z	$x \to y^c \le z^c$	iff	$x \leq y > z^c$.			

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x = y = z	$x \to y^c \le z^c$	iff	$x \leq y > z^c$.			

Definition (Restricting inequality)

Restricting inequalities are inequalities of shape

$$\mathbf{i} \leq f(\bar{\mathbf{j}},\overline{\mathbf{m}}), \qquad g(\overline{\mathbf{m}},\bar{\mathbf{j}}) \leq \mathbf{n}, \qquad \mathbf{i} \leq \mathbf{h}, \qquad \mathbf{o} \leq \mathbf{n}.$$

Restricted quantifiers

Flattification

 $\mathbf{j} \leq f(\overline{\alpha}, \overline{\beta}) \quad \text{iff} \quad (\exists \overline{\mathbf{i}}, \overline{\mathbf{n}} \triangleright_f \mathbf{j})(\& \mathbf{i} \leq \alpha \& \& \beta \leq \mathbf{n})$ $f(\overline{\alpha}, \overline{\beta}) \leq \mathbf{m} \quad \text{iff} \quad (\forall \overline{\mathbf{i}}, \overline{\mathbf{n}} \triangleright_f \lambda \mathbf{m})(? \alpha \leq \kappa \mathbf{i} ? ? ? \lambda \mathbf{n} \leq \beta)$ $\mathbf{j} \leq g(\overline{\alpha}, \overline{\beta}) \quad \text{iff} \quad (\forall \overline{\mathbf{n}}, \overline{\mathbf{i}} \triangleright_g \kappa \mathbf{j})(? \lambda \mathbf{n} \leq \alpha ? ? ? \beta \leq \kappa \mathbf{i})$ $g(\overline{\alpha}, \overline{\beta}) \leq \mathbf{m} \quad \text{iff} \quad (\exists \overline{\mathbf{n}}, \overline{\mathbf{i}} \triangleright_g \mathbf{m})(\& \alpha \leq \mathbf{n} \& \& \mathbf{i} \leq \beta)$

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Conclusions

Main contributions:

▶ Sahlqvist \rightsquigarrow very simple Sahlqvist in $\mathcal{L}^* \supseteq$ inductive

- Boolean ~> distributive
- semantic and signature-agnostic

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Thank you!

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