Peter Arndt, jt. with Hugo Mariano and Darllan Pinto

TACL 2022

Plan:

- Categories of logics and translations
- emote algebraizability
- More on categories of logics and translations
- Algebraizability as an algebraic structure
- Surther points

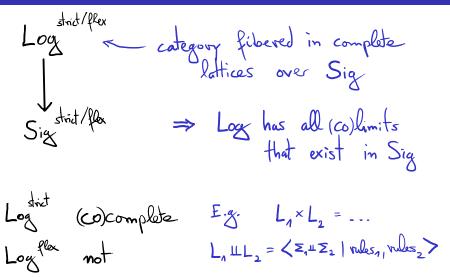
Signature :
$$\Sigma = (\Sigma_n)_{n \in \mathbb{N}}$$

 $\longrightarrow F_{m_{\Sigma}}(X)$ formulas
Strict morphism $\Sigma \xrightarrow{f} \Sigma'$: $(f_n : \Sigma_n \rightarrow \Sigma'_n)_{n \in \mathbb{N}}$
Flexible morphism : $(f_n : \Sigma_n \rightarrow F_{m_{\Sigma'}}(X)[n])_{n \in \mathbb{N}}$

Logic:
$$L = (\Sigma, F)$$

signature finitary, structural
cons. vel. on $\overline{Im}_{\Sigma}(X)$
these form a complete lattice
in $\Im(\Im(\overline{Im}_{\Sigma}(X)) \times \overline{Im}_{\Sigma}(X))$
 $\Rightarrow \cdot \operatorname{can}$ generate a cons. vel. from rules
 $\cdot \overline{Im}_{\Sigma}(X) \xrightarrow{\rightarrow} \overline{Im}_{\Sigma^{1}}(X)$
 $p_{\Sigma}^{*}(F) \xleftarrow{} F_{\Sigma}(X)$

Strict / flexible translation
$$(\Sigma, H) \longrightarrow (\Sigma', H')$$
:
strict / flex. signature morph. $\Sigma \xrightarrow{f} \Sigma'$
s.t. $\Gamma H \varphi \implies f(\Gamma) H f(\varphi)$
Equivalent: $H \subseteq f^{*}(H)$
 $f_{*}(H) \subseteq H$
 \longrightarrow categories Locg , Logflex



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Possible Translations Semantics (Carnielli):

$$\{L \xrightarrow{k} L_i \mid i \in I\}$$
 s.t. $\Gamma \vdash \Psi \iff \forall i : f_i(\Gamma) \vdash f_i(\Psi)$
Equivalently: $\vdash = \bigwedge_{i \in I} f_i^*(\vdash_i)$
Equivalently: $L \xrightarrow{\langle f_i \rangle} TTL_i$ is conservative
(i.e. reflects deductions)

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L = (Z,F) is protodybraic : ⇐⇒ I set of binary connectives = { ->, ..., ->, } S.t. & + x -> , x and {x, x -> , y ..., x -> , y }+ y L is n-protoalgebraic if (=> (≤n. Generic n-protoalgebraic logic: $P_n := \left(\{ \neg_{1}, ..., \neg_n \}, F \right) \quad \text{by those}$ $\left[L \text{ is } n \text{- protoalgebraic iff } \exists P_n \rightarrow L \right]$ P. Andt

Protoalgebraie logics are not closed under products: TTPn does not satisfy Modus ponens (finitary) ... but under finite products. L remotely protoalgebraic via finite family ⇐ = conservative translation L -> L' an n-protoalg. logic ⇐ coproduct inclusion L→LILPn ← Pn is conservative

L is finitely algebraizable if
$$\exists \text{ binary formulas } \Delta = \{\Delta_{n1}, \Delta_{k}\}$$

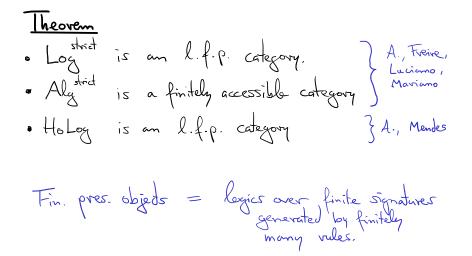
and unarry formulas $\langle \delta_{1} \varepsilon \rangle = \{\delta_{n1} \varepsilon_{n1}, \dots, \delta_{l1} \varepsilon_{l2}\}$ s.t.
(i) $\vdash s \varphi \Delta \varphi$;
(ii) $\varphi \Delta \psi \vdash s \psi \Delta \varphi$;
(iii) $\varphi \Delta \psi, \psi \Delta \vartheta \vdash s \varphi \Delta \vartheta$;
For every $n - ary$ connective ω
(iv) $\varphi_{0} \Delta \psi_{0}, \dots, \varphi_{n-1} \Delta \psi_{n-1} \vdash s \omega \varphi_{0} \dots \varphi_{n-1} \Delta \omega \psi_{0} \dots \psi_{n-1}$.
(v) $\vartheta \dashv s \delta(\vartheta) \Delta \epsilon(\vartheta)$.
L is (k,l) -algebraizable if $|\Delta| = k$, $|\langle \delta_{1} \varepsilon \rangle| = 2l$
Generic (k,l) -algebraizable $\log c$: $A_{k,l}$
 $[\exists A_{k,l} \longrightarrow L] \not\approx L (k,l)$ -algebraizable]

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A diagram is fibered if
$$\forall A_1 = B$$

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Have functors
$$A_{k,e}$$
: Holog \longrightarrow Holog
 $L \longmapsto L \otimes A_{k,e}$
translations: $L \longrightarrow L \otimes A_{k,e}$
 $L \otimes A_{k,e} \otimes A_{k,e} \longrightarrow L \otimes A_{k,e}$

Theorem: Akie-Alg is the category of (kil)-djebraizable logics and translations that preserve algebraizing pairs Kroof: L Ake-algebra $L \otimes A_{k,k} \xrightarrow{f \otimes i \partial} L' \otimes A_{k,k} \xrightarrow{f} preserves decorraising}$ $L \xrightarrow{L'} \xrightarrow{L'} \xrightarrow{f \otimes i \partial} L' \xrightarrow{$ \Box

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Further points

Poset of all logics

Comparison with the poset of all bogics (Jonsona/Moraschini)
An interpretation
$$L \stackrel{f}{\rightarrow} L'$$
 is a (see finiting here!)
plex. translation s.t. $Mod(L') \longrightarrow Mod(L)$
(A,F) $\longmapsto A^{f},F$)
sends reduced matrices to reduced advect the
matrices. along the
signature morphism.
mod category Int of logics and interpretations
Poset of all logics := poset reflection of Int

Z. M .: The posst of all logics has (now also infinitary) set-indexed infima, but in general no finite suprema. (Compare initial logic)

Poset reflection of Hology is a complete lattice

Poset of all logics

JM: fin. pres. Leibniz class: Sequence of fin. pres. equivalential logics {Li}ieN Denseness not necessary] This work suggests: fin pres. Leibniz classes correspond to accessible subcategories of HoLog.

Further Thoughts

Thanks !